

Random subgraphs

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This talk: G is a general finite graph.

Typically = asymptotically almost surely (a.a.s.)

Properties of $G(n, p)$

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The main difficulty is that this is **not the right statement to prove**.

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[Christoph-Morawski-W. 2026]

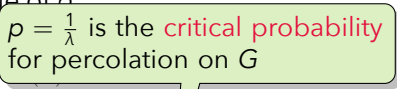
If $p \geq \frac{1+\varepsilon}{d}$, then a.a.s. G_p has a component of order $\Omega_\varepsilon(d)$.

Theorem

[Christoph-Morawski-W. 2026]

If $p \geq \frac{1+\varepsilon}{\lambda}$, then a.a.s. G_p has a component of order $\Omega_\varepsilon(\lambda)$.

Good: $\lambda \geq d$ always (equality iff G is d -regular), so this is a larger component with a smaller value of p .

Bad: "almost asymptotically all" probability 99.99% but not 1 -  $p = \frac{1}{\lambda}$ is the critical probability for percolation on G

This restriction is necessary in general, but can be boosted to a.a.s. except in a very specific regime: $\lambda = C\sqrt{\Delta}$.

$\exists$ matching subcritical statements: if $p \leq \frac{1-\varepsilon}{\lambda}$ then G_p typically has all components of order $o(\lambda)$... but major subtleties if $\lambda = C\sqrt{\Delta}$.

Proof ideas

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How to analyze G_p ? [Krivelevich–Sudakov]

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- Explore G_p one edge at a time.

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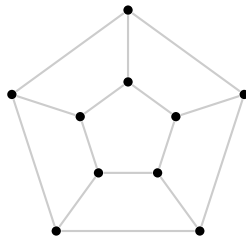
How to analyze G_p ? [Krivelevich-Sudakov]

- Explore G_p one edge at a time.
- Every time we query a new edge, we flip a $\text{Ber}(p)$ coin.

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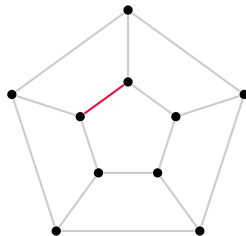
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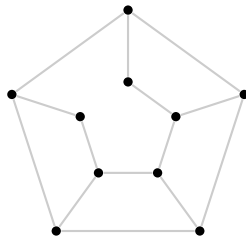
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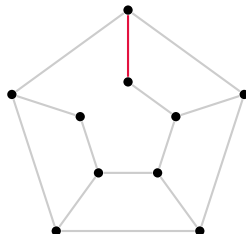
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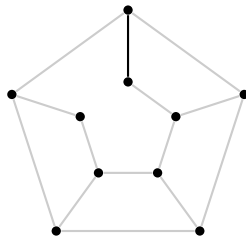
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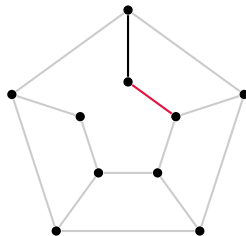
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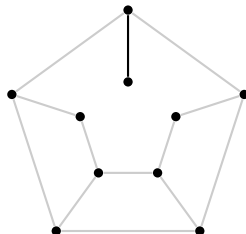
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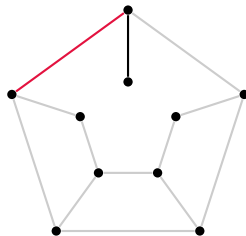
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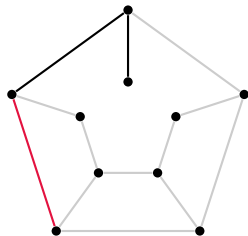
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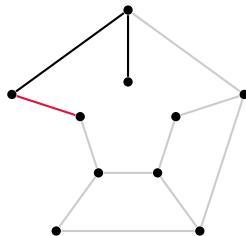
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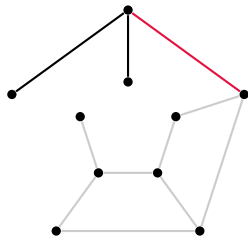
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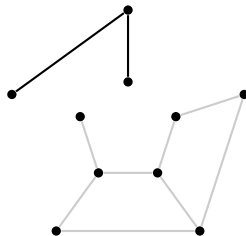
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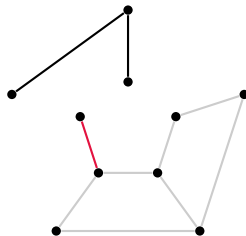
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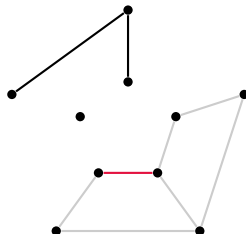
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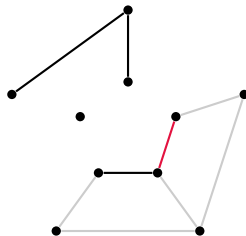
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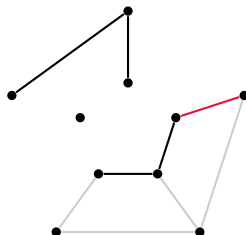
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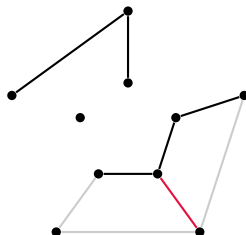
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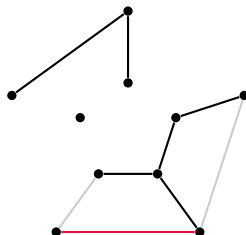
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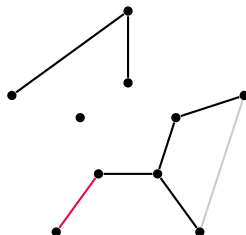
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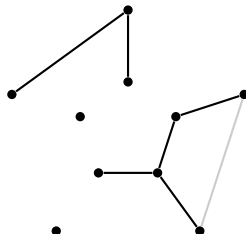
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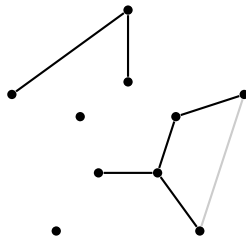
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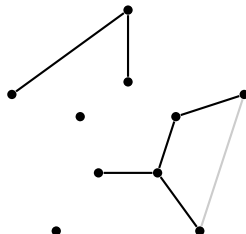
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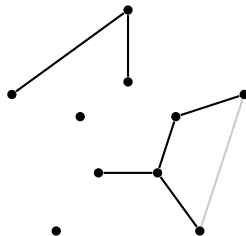
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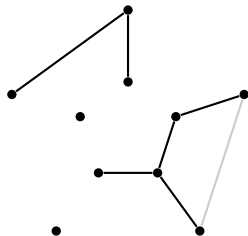


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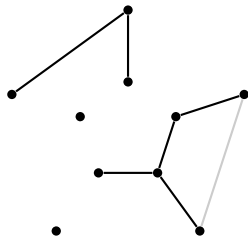


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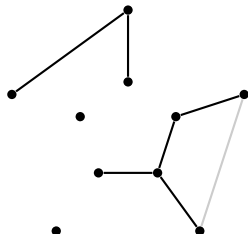


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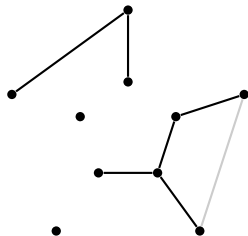
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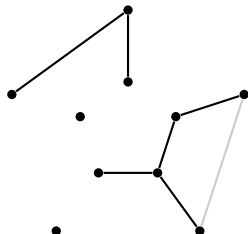
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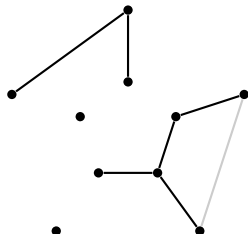
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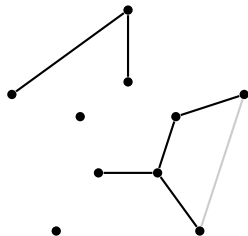
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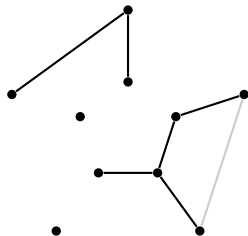
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Using the eigenvector as a weight vector solves **both** problems.

Thank you!